Superfluid Weight vs. Superconducting Temperature based on a U(1) Slave-Boson Approach to the t-J Hamiltonian

Sung-Sik Lee and Sung-Ho Suck Salk

Department of Physics, Pohang University of Science and Technology,

Pohang, Kyoungbuk, Korea 790-784

(February 1, 2008)

Abstract

Based on an improved U(1) slave-boson approach to the t-J Hamiltonian, we investigate a relationship between the superfluid weight n_s/m^*(the superconducting charge carrier density/the effective mass of the charge carrier) and the superconducting temperature T_c. From the present study we find a linear increase of n_s/m^* with T_c with the doping concentration in the underdoped region, a saturation around the optimal doping and a decrease in both n_s/m^* and T_c in the overdoped region. Such a trend of the 'boomerang' shaped locus in n_s/m^* vs. T_c with increasing doping concentration from the underdoped to the heavily overdoped region is predicted to be in complete agreement with muon-spin-relaxation measurements. The boomerang behavior is found to occur in correlation with reduction in the spin singlet pairing(spinon pairing) order in the heavily overdoped region.

The transverse field muon-spin-relaxation(μ-SR) measurements of the magnetic penetration depth λ in high T_c copper oxide superconductors reveal an universal linear increase of the superfluid weight n_s/m^*(the superconducting charge carrier density / the effective mass of the charge carrier) with the superconducting transition temperature T_c in the underdoped region, a saturation near the optimal doping and a decrease in both n_s/m^* and T_c as the hole doping concentration increases from the underdoped region to the heavily overdoped region [1] [2] [3]. In the underdoped region the observed linear increase in both n_s/m^* and T_c scales well with the doped hole concentration p while n_s/m^* vs. T_c does not scale with p near and over the optimal doping. One can conjecture that the observed reduction of n_s/m^* with increasing hole carrier concentration in the overdoped region is attributed to either a decrease in the superconducting charge carrier density n_s or to an increase in the effective mass of the charge carrier m^*. In the present study we report a study of the observed boomerang behavior [2] [3] in n_s/m^* vs. T_c as a function of doped hole concentration by using an improved U(1) slave-boson theory which correctly allows coupling between the spin and charge degrees of freedom. Contrary to our earlier U(1) slave-boson approach [4], such introduction of both the spin and charge degrees of freedom was found to predict the arch shaped bose condensation temperature as a function of doped hole concentration p [5], in agreement with the experimentally observed phase diagram in the plane of T_c and p. For this study we first compute the doping and temperature dependences of the superfluid weight and discuss the cause of reduction in both the superfluid weight and the superconducting temperature in the overdoped region. In addition, a numerical comparison with the SU(2) slave-boson theory is briefly made to discuss a difference.

We write the t-J Hamiltonian,

$$H = -t \sum_{<i,j>} (c_{i\sigma}^\dagger c_{j\sigma} + c.c.) + J \sum_{<i,j>} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j) - \mu_0 \sum_{i,\sigma} c_{i\sigma}^\dagger c_{i\sigma}, \tag{1}$$

where $\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j = -\frac{1}{2}(c_{i\downarrow}^\dagger c_{j\uparrow}^\dagger - c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger)(c_{j\uparrow} c_{i\downarrow} - c_{j\downarrow} c_{i\uparrow})$. Here $\mathbf{S}_i$ is the electron spin operator at site i, $\mathbf{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta}$ with $\boldsymbol{\sigma}_{\alpha\beta}$, the Pauli spin matrix element and n_i, the electron number operator at site i, $n_i = c_{i\sigma}^\dagger c_{i\sigma}$. Using the single occupancy constraint and thus $c_{i\sigma} = b_i^\dagger f_{i\sigma}$(with $f_{i\sigma}$, spinon annihilation operator of electron spin σ and $b_i^\dagger$, holon creation operator at site i), the U(1) slave-boson representation of the above t-J Hamiltonian leads to Eq.(2),

$$H = -t \sum_{<i,j>} (f_{i\sigma}^\dagger f_{j\sigma} b_j^\dagger b_i + c.c.)$$

$$-\frac{J}{2}\sum_{<i,j>} b_i b_j b_j^\dagger b_i^\dagger (f_{\downarrow i}^\dagger f_{\uparrow j}^\dagger - f_{\uparrow i}^\dagger f_{\downarrow j}^\dagger)(f_{\uparrow j} f_{\downarrow i} - f_{\downarrow j} f_{\uparrow i})$$

$$-\mu_0 \sum_{i,\sigma} f_{i\sigma}^\dagger f_{i\sigma} + i\sum_i \lambda_i (f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i - 1), \tag{2}$$

where λ_i is the Lagrange multiplier field to enforce the single occupancy constraint at each site. In the SU(2) slave-boson theory, the electron operator is given by $c_\alpha = \frac{1}{\sqrt{2}} h^\dagger \psi_\alpha$ with $\alpha = 1, 2$, where $\psi_1 = \begin{pmatrix} f_1 \\ f_2^\dagger \end{pmatrix}$ and $\psi_2 = \begin{pmatrix} f_2 \\ -f_1^\dagger \end{pmatrix}$ and $h = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ are respectively the doublets of spinon and holon annihilation operators in the SU(2) theory. The SU(2) slave-boson representation of the above t-J Hamiltonian shows

$$H = -\frac{t}{2} \sum_{<i,j>\sigma} \left[(f_{\sigma i}^\dagger f_{\sigma j})(b_{1j}^\dagger b_{1i} - b_{2i}^\dagger b_{2j}) \right.$$

$$+ (f_{\sigma j}^\dagger f_{\sigma i})(b_{1i}^\dagger b_{1j} - b_{2j}^\dagger b_{2i})$$

$$+ (f_{2i} f_{1j} - f_{1i} f_{2j})(b_{1j}^\dagger b_{2i} + b_{1i}^\dagger b_{2j})$$

$$\left. + (f_{1j}^\dagger f_{2i}^\dagger - f_{2j}^\dagger f_{1i}^\dagger)(b_{2i}^\dagger b_{1j} + b_{2j}^\dagger b_{1i}) \right]$$

$$- \frac{J}{2} \sum_{<i,j>} (1 - h_i^\dagger h_i)(1 - h_j^\dagger h_j) \times$$

$$(f_{2i}^\dagger f_{1j}^\dagger - f_{1i}^\dagger f_{2j}^\dagger)(f_{1j} f_{2i} - f_{2j} f_{1i}) - \mu_0 \sum_i h_i^\dagger h_i$$

$$- \sum_i \left[i\lambda_i^{(1)}(f_{1i}^\dagger f_{2i}^\dagger + b_{1i}^\dagger b_{2i}) + i\lambda_i^{(2)}(f_{2i} f_{1i} + b_{2i}^\dagger b_{1i}) \right.$$

$$\left. + i\lambda_i^{(3)}(f_{1i}^\dagger f_{1i} - f_{2i} f_{2i}^\dagger + b_{1i}^\dagger b_{1i} - b_{2i}^\dagger b_{2i}) \right], \tag{3}$$

where $\lambda_i^{(1),(2),(3)}$ are the real Lagrangian multipliers to enforce the local single occupancy constraint in the SU(2) slave-boson representation [9]. It is noted that in both the U(1) and SU(2) approaches above, coupling between spin and charge degrees of freedom is correctly introduced in the Heisenberg term.

After Hubbard Stratonovich transformations for the direct, exchange and pairing channels, we find the total free energy in the functional integral representation, for the U(1) slave-boson theory [5],

$$F(\mathbf{A}) = -\frac{1}{\beta} \ln \int D\chi D\Delta^f D\Delta^b Db Df$$

$$e^{-(S^b(\mathbf{A},\chi,\Delta^f,\Delta^b,b) + S^f(\chi,\Delta^f,f))}, \tag{4}$$

where S^b represents the holon action for the charge degree of freedom,

$$S^b(\mathbf{A}, \chi, \Delta^f, \Delta^b, b) = \int_0^\beta d\tau \Big[\sum_i b^\dagger(\mathbf{r}_i, \tau)(\partial_\tau - \mu^b)b(\mathbf{r}_i, \tau)$$

$$+ \frac{J}{2} \sum_{<i,j>} |\Delta^f_{ij}|^2 \Big[|\Delta^b_{ij}|^2 + p^2 \Big]$$

$$- t \sum_{<i,j>} e^{iA_{ij}} \chi_{ij} b^\dagger(\mathbf{r}_i, \tau) b(\mathbf{r}_j, \tau) + c.c.$$

$$- \frac{J}{2} \sum_{<i,j>} |\Delta^f_{ij}|^2 \Big[\Delta^b_{ij} b^\dagger(\mathbf{r}_i, \tau) b^\dagger(\mathbf{r}_j, \tau) + c.c. \Big] \tag{5}$$

and S^f is the spinon action for the spin degrees of freedom,

$$S^f(\chi, \Delta^f, f) = \int_0^\beta d\tau \Big[\sum_i f^\dagger_\sigma(\mathbf{r}_i, \tau)(\partial_\tau - \mu^f) f_\sigma(\mathbf{r}_i, \tau)$$

$$+ \frac{J(1-p^2)}{2} \sum_{<i,j>} \Big[|\Delta^f_{ij}|^2 + \frac{1}{2}|\chi_{ij}|^2 \Big]$$

$$- \frac{J}{4}(1-p)^2 \sum_{<i,j>} \chi_{ij} f^\dagger_\sigma(\mathbf{r}_i, \tau) f_\sigma(\mathbf{r}_j, \tau) + c.c.$$

$$- \frac{J}{2}(1-p)^2 \sum_{<i,j>} \Delta^f_{ij}(f_\downarrow(\mathbf{r}_i, \tau) f_\uparrow(\mathbf{r}_j, \tau) - f_\uparrow(\mathbf{r}_i, \tau) f_\downarrow(\mathbf{r}_j, \tau)) + c.c. \Big]. \tag{6}$$

Here b is the holon field and f, the spinon field. χ_{ij}, Δ^f_{ij} and Δ^b_{ij} are the Hubbard Stratonovich fields corresponding to the exchange, the spinon pairing and the holon pairing channels respectively. After integration over the holon and spinon fields, we obtain the total free energy,

$$F(\mathbf{A}) = -\frac{1}{\beta} \ln \int D\chi D\Delta^f D\Delta^b e^{-(F^b(\mathbf{A}, \chi, \Delta^f, \Delta^b) + F^f(\chi, \Delta^f))}, \tag{7}$$

where $F^b(\mathbf{A}, \chi, \Delta^f, \Delta^b) = -\frac{1}{\beta} \ln \int Db\, e^{-S^b(\mathbf{A}, \chi, \Delta^f, \Delta^b, b)}$ is the holon free energy and $F^f(\chi, \Delta^f) = -\frac{1}{\beta} \ln \int Df\, e^{-S^f(\chi, \Delta^f, f)}$, the spinon free energy.

The linear response of current to weak applied electromagnetic(EM) field is, in the energy-momentum space,

$$j_l(\omega, \mathbf{q}) = -\Pi_{lm}(\omega, \mathbf{q}) A_m(\omega, \mathbf{q}), \tag{8}$$

with $\mathbf{j}$, the current and $\mathbf{A}$, the EM vector potential. Here the current response function $\Pi_{lm}(\omega, \mathbf{q})$ is obtained from

$$\Pi_{lm}(\omega, \mathbf{q}) = -\beta \frac{\partial^2}{\partial A_l(-\omega, -\mathbf{q}) \partial A_m(\omega, \mathbf{q})} F(\mathbf{A}) \Big|_{\mathbf{A}=0}, \tag{9}$$

4

with $\beta = \frac{1}{k_B T}$, where $F(\mathbf{A})$ is the free energy of Eq.(7) or Eq.(??). The superfluid weight $\frac{n_s}{m^*}$ is defined as the transverse EM current response function in the static, long wave length limit [6],

$$\frac{n_s}{m^*} = \frac{1}{e^2} \lim_{q \to 0} \Pi_{xx}(\omega = 0, \mathbf{q}), \tag{10}$$

for the isotropic system.

As is shown in the second term of Eq.(5), the EM vector potential field A is seen to modulate(is coupled with) the hopping order parameter field χ_{ij} in the U(1) slave-boson representation. The phase fluctuations of the hopping order parameter, i.e. $\chi_{ij} = \chi e^{i a_{ij}}$ are introduced. For the hole pairing of d-wave symmetry we allow the d-wave spinon pairing order parameter, $\Delta_{ji}^f = \pm \Delta_f$(the sign $+(-)$ is for the **ij** link parallel to $\hat{x}$ ($\hat{y}$)) and the s-wave holon pairing order parameter, $\Delta_{ji}^b = \Delta_b$. The EM current response function of the total system is obtained from the Ioffe-Larkin composition rule [11],

$$\Pi(\omega, \mathbf{q}) = \frac{\Pi^b(\omega, \mathbf{q}) \Pi^f(\omega, \mathbf{q})}{\Pi^b(\omega, \mathbf{q}) + \Pi^f(\omega, \mathbf{q})}, \tag{11}$$

where $\Pi^b(\omega, \mathbf{q})$ is the holon current response function to the the EM field A and $\Pi^f(\omega, \mathbf{q})$, the spinon current response function to the gauge field a corresponding to the phase fluctuations of hopping order parameter. The holon and spinon response functions are calculated from the linear response theory(see appendices 1,2). The holon response function is evaluated to be

$$
\begin{aligned}
\Pi_{lm}^b(\omega, \mathbf{q}) = \frac{1}{N} \sum_{\mathbf{q_1}} \Bigg[& -2t\chi \cos q_{1l}(u^b(\mathbf{q_1})^2 n^b(E^b(\mathbf{q_1})) - v^b(\mathbf{q_1})^2 n^b(-E^b(\mathbf{q_1}))) \delta_{lm} \\
& + 4t^2\chi^2 e^{i\frac{q_l - q_m}{2}} \sin(q_{1m} + \frac{q_m}{2}) \sin(q_{1l} + \frac{q_l}{2}) \times \\
& \Bigg\{ (u^b(\mathbf{q}+\mathbf{q_1})^2 u^b(\mathbf{q_1})^2 - u^b(\mathbf{q}+\mathbf{q_1})u^b(\mathbf{q_1})v^b(\mathbf{q}+\mathbf{q_1})v^b(\mathbf{q_1})) \frac{n^b(E^b(\mathbf{q}+\mathbf{q_1})) - n^b(E^b(\mathbf{q_1}))}{i\omega - (E^b(\mathbf{q}+\mathbf{q_1}) - E^b(\mathbf{q_1}))} \\
& + (-u^b(\mathbf{q}+\mathbf{q_1})^2 v^b(\mathbf{q_1})^2 + u^b(\mathbf{q}+\mathbf{q_1})u^b(\mathbf{q_1})v^b(\mathbf{q}+\mathbf{q_1})v^b(\mathbf{q_1})) \frac{n^b(E^b(\mathbf{q}+\mathbf{q_1})) - n^b(-E^b(\mathbf{q_1}))}{i\omega - (E^b(\mathbf{q}+\mathbf{q_1}) + E^b(\mathbf{q_1}))} \\
& + (-v^b(\mathbf{q}+\mathbf{q_1})^2 u^b(\mathbf{q_1})^2 + u^b(\mathbf{q}+\mathbf{q_1})u^b(\mathbf{q_1})v^b(\mathbf{q}+\mathbf{q_1})v^b(\mathbf{q_1})) \frac{n^b(-E^b(\mathbf{q}+\mathbf{q_1})) - n^b(E^b(\mathbf{q_1}))}{i\omega + (E^b(\mathbf{q}+\mathbf{q_1}) + E^b(\mathbf{q_1}))} \\
& + (v^b(\mathbf{q}+\mathbf{q_1})^2 v^b(\mathbf{q_1})^2 - u^b(\mathbf{q}+\mathbf{q_1})u^b(\mathbf{q_1})v^b(\mathbf{q}+\mathbf{q_1})v^b(\mathbf{q_1})) \frac{n^b(-E^b(\mathbf{q}+\mathbf{q_1})) - n^b(-E^b(\mathbf{q_1}))}{i\omega + (E^b(\mathbf{q}+\mathbf{q_1}) - E^b(\mathbf{q_1}))} \Bigg\} \Bigg], \tag{12}
\end{aligned}
$$

where $E^b(\mathbf{q}) = \sqrt{(\epsilon^b(\mathbf{q}) - \mu^b)^2 - (J\Delta_f^2 \Delta_b \gamma_\mathbf{q})^2}$ is the holon quasiparticle energy and $\epsilon^b(\mathbf{q}) = -2t\chi\gamma_\mathbf{q}$, the single holon quasiparticle energy with $\gamma_\mathbf{q} = (\cos q_x + \cos q_y)$. $n^b(E) = \frac{1}{e^{\beta E} - 1}$ is the boson distribution function, $u^b(\mathbf{q}) = \frac{1}{\sqrt{2}} \sqrt{\frac{\epsilon^b(\mathbf{q}) - \mu^b}{E^b(\mathbf{q})} + 1}$ and $v^b(\mathbf{q}) = \frac{1}{\sqrt{2}} \sqrt{\frac{\epsilon^b(\mathbf{q}) - \mu^b}{E^b(\mathbf{q})} - 1}$. The spinon response function is

$$\Pi_{lm}^f(\omega,\mathbf{q}) = \frac{1}{N}\sum_{\mathbf{q}_1}\Bigg[-\frac{J}{2}(1-p)^2\chi\cos q_{1l}\frac{\epsilon^f(\mathbf{q}_1)-\mu^f}{E^f(\mathbf{q}_1)}(n^f(E^f(\mathbf{q}_1))-n^f(-E^f(\mathbf{q}_1)))\delta_{lm}$$

$$+(\frac{J}{2}(1-p)^2\chi)^2 e^{i\frac{q_l-q_m}{2}}\sin(q_{1m}+\frac{q_m}{2})\sin(q_{1l}+\frac{q_l}{2})\times$$

$$\Bigg\{(n^f(-E^f(\mathbf{q}+\mathbf{q}_1))\frac{E^f(\mathbf{q}+\mathbf{q}_1)(i\omega+E^f(\mathbf{q}+\mathbf{q}_1))+(\epsilon^f(\mathbf{q}+\mathbf{q}_1)-\mu^f)(\epsilon^f(\mathbf{q}_1)-\mu^f)+\Delta_f'(\mathbf{q}+\mathbf{q}_1)\Delta_f'(\mathbf{q}_1)}{E^f(\mathbf{q}+\mathbf{q}_1)[(i\omega+E^f(\mathbf{q}+\mathbf{q}_1))^2-E^f(\mathbf{q}_1)^2]}$$

$$+(n^f(-E^f(\mathbf{q}_1))\frac{E^f(\mathbf{q}_1)(-i\omega+E^f(\mathbf{q}_1))+(\epsilon^f(\mathbf{q}+\mathbf{q}_1)-\mu^f)(\epsilon^f(\mathbf{q}_1)-\mu^f)+\Delta_f'(\mathbf{q}+\mathbf{q}_1)\Delta_f'(\mathbf{q}_1)}{E^f(\mathbf{q}_1)[(-i\omega+E^f(\mathbf{q}_1))^2-E^f(\mathbf{q}+\mathbf{q}_1)^2]}$$

$$-(n^f(E^f(\mathbf{q}+\mathbf{q}_1))\frac{E^f(\mathbf{q}+\mathbf{q}_1)(-i\omega+E^f(\mathbf{q}+\mathbf{q}_1))+(\epsilon^f(\mathbf{q}+\mathbf{q}_1)-\mu^f)(\epsilon^f(\mathbf{q}_1)-\mu^f)+\Delta_f'(\mathbf{q}+\mathbf{q}_1)\Delta_f'(\mathbf{q}_1)}{E^f(\mathbf{q}+\mathbf{q}_1)[(-i\omega+E^f(\mathbf{q}+\mathbf{q}_1))^2-E^f(\mathbf{q}_1)^2]}$$

$$-(n^f(E^f(\mathbf{q}_1))\frac{E^f(\mathbf{q}_1)(i\omega+E^f(\mathbf{q}_1))+(\epsilon^f(\mathbf{q}+\mathbf{q}_1)-\mu^f)(\epsilon^f(\mathbf{q}_1)-\mu^f)+\Delta_f'(\mathbf{q}+\mathbf{q}_1)\Delta_f'(\mathbf{q}_1)}{E^f(\mathbf{q}_1)[(i\omega+E^f(\mathbf{q}_1))^2-E^f(\mathbf{q}+\mathbf{q}_1)^2]}\Bigg\}\Bigg], \qquad (13)$$

where $E^f(\mathbf{q}) = \sqrt{(\epsilon^f(\mathbf{q})-\mu^f)^2+(\Delta_f'(\mathbf{q}))^2}$ is the spinon quasiparticle energy, $\epsilon^f(\mathbf{q}) = -\frac{J}{2}(1-p)^2\chi\gamma_{\mathbf{q}}$, the single spinon quasiparticle energy, and $\Delta_f'(\mathbf{q}) = J(1-p)^2\Delta_f\varphi_{\mathbf{q}}$, the spinon pairing gap with $\varphi_{\mathbf{q}} = (\cos q_x - \cos q_y)$. $n^f(E) = \frac{1}{e^{\beta E}+1}$ is the fermion distribution function. For the U(1) theory the superfluid weight is obtained from Eq.(10) with the use of (11). In the present calculations of the superfluid weight $J/t = 0.2$ is chosen. For other choice of J/t we find that there exist no qualitative differences in the behavior of n_s/m^* vs. T_c.

In Fig.1 the computed superfluid weight displays a linear increase in doped hole concentration(rate) p in the underdoped region(the predicted optimal doping rate is $p_o = 0.07$). As hole doping concentration further increases, the superfluid weight first saturates and rapidly drops to 0 in the heavily overdoped region. This behavior in the overdoped region is attributed to a decrease in holon pairing interaction caused by the predicted diminishing trend of the spinon pairing probability amplitude(order parameter) Δ_{ij}^f, as can be readily understood from the last term of Eq.(5). The predicted decrease of pseudogap(spin gap) temperature with hole doping rate is in excellent agreement with observation. In Fig.2, a "boomerang" shaped locus in $\frac{n_s(T\to 0)}{m^*}$ and T_c is displayed by showing a linear relationship between the two in the underdoped region and the 'reflex' behavior in the overdoped region. Although not numerically agreeable, this trend is completely consistent with muon-spin-relaxation measurements [2] [3]. As mentioned above, the spinon pairing amplitude Δ_{ij}^f is further reduced in the overdoped region and this, in turn, causes a decrease in holon pairing interaction and thus the superconducting charge carrier density n_s. This will allow the reflex behavior, that is, a decrease in both the superfluid weight and superconducting temperature.

For the sake of introducing the low energy phase fluctuations of the order parameters χ_{ij} and Δ_{ij}^f,

which were not considered in the above U(1) slave-boson study, we now introduce the SU(2) slave-boson theory. In the SU(2) theory, the EM field A is found to modulate both the hopping order parameter χ_{ij} and the spinon pairing order parameter Δ_{ij}^f. The phase fluctuations of both χ_{ij} and Δ_{ij}^f are thus taken into account to assess the EM current response of the system. We introduce the spinon pairing order parameter of d-wave symmetry, $\Delta_{ji}^f = \pm\Delta_f$(the sign $+(-)$ is for the **ij** link parallel to $\hat{x}$ $(\hat{y})$) and the holon pairing order parameter of s-wave symmetry, $\Delta_{ij;\alpha\beta}^B = \Delta_b(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2})$.

In the SU(2) slave-boson theory, the response function $\Pi_{ll'}^{b(A,a^1)}$ of holon isospin current to the EM field A and the gauge field a vanishes owing to the contribution of the b_2-boson in the static and long-wavelength limit(see the appendices 3,4). Therefore the superfluid weight of the total system is given by the holon current response function only,

$$\frac{n_s}{m^*} = \frac{1}{e^2}\Pi_{lm}^{b(A,A)}(\omega = 0, \mathbf{q} = 0). \tag{14}$$

Here $\Pi_{lm}^{b(A,A)}(\omega, \mathbf{q})$ is computed from the use of the usual linear response theory for the holon action Eq.(**??**)(see Appendix 6). In Fig.3, we show the doping dependence of the superfluid weight with the choice of $J/t = 0.2$. The superfluid weight predicted from the SU(2) theory increases faster than the U(1) case, as doped hole concentration. This is because the gauge fields(or the phase fluctuations of the order parameters) do not screen the EM field in the SU(2) theory. In Fig.4, the relation between T_c and $\frac{n_s(T\rightarrow 0)}{m^*}$ is displayed; at low doping, T_c increases linearly with $\frac{n_s(T\rightarrow 0)}{m^*}$ as in the U(1) theory, a plateau near the optimal doping $p = 0.13$) and a reflex in the overdoped region is also observed. There exists only a quantitative (but not qualitative) difference between the two theories. This is because the spinon pairing amplitude Δ_{ij}^f predicted by the SU(2) theory slowly diminishes in the overdoped region compared to the U(1) case.

In summary, we investigated a relationship between the superfluid weight and the superconducting temperature, that is, $\frac{n_s}{m^*}$ vs. T_c based on both the U(1) and SU(2) slave-boson approaches to the t-J Hamiltonian. From both theories we find a qualitatively similar boomerang shape in the path of $\frac{n_s(T\rightarrow 0)}{m^*}$ vs. T_c by showing a linear increase in the underdoped region and a reflex behavior in the overdoped region. Such trend of boomerang behavior in high T_c superconductors is completely consistent with μ-SR experiments. The reflex behavior predicted by both theories is attributed to the diminishing attractive hole pairing interaction caused by the markedly reduced spin pairing order Δ^f, particularly in the heavily overdoped region, and thus to the reduction of both the superconducting charge carrier density n_s and the superconducting temperature T_c in the overdoped region.

REFERENCES

[1] Y. J. Uemura et al, Phys. Rev. Lett. **62**, 2317 (1989); Y. J. Uemura et al, Phys. Rev. Lett. **66**, 2665 (1991).

[2] Y. J. Uemura, A. Keren, L. P. Le, G. M. Luke, W. D. Wu, Y. Kubo, T. Manako, Y. Shimakawa, M. Subramanian, J. L. Cobb and J. T. Markert, Nature **364**, 605 (1993).

[3] C. Bernhard, Ch. Niedermayer, U. Binninger, A. Hofer, Ch. Wenger, J. L. Tallon, G. V. M. Williams, E. J. Ansaldo, J. I. Budnick, C. E. Stronach, D. R. Noakes, and M. A. Blankson-Mills, Phys. Rev. B **52**, 10488 (1995); references therein.

[4] T.-H. Gimm, S.-S. Lee, S.-P. Hong and Sung-Ho Suck Salk, Phys. Rev. B **60**, 6324 (1999).

[5] S.-S. Lee and Sung-Ho Suck Salk, Int. J. Mod. Phys. **13**, 3455 (1999)(cond-mat/9905268).

[6] E. Dagotto, Rev. Mod. Phys. **60**, 763 (1994).

[7] G. Kotliar and J. Liu, Phys. Rev. B **38**, 5142 (1988); references there-in.

[8] a) M. U. Ubbens and P. A. Lee, Phys. Rev. B **46**, 8434 (1992); b) M. U. Ubbens and P. A. Lee, Phys. Rev. B **49**, 6853 (1994); references there-in.

[9] a) X. G. Wen and P. A. Lee, Phys. Rev. Lett. **76**, 503 (1996); b) X. G. Wen and P. A. Lee, Phys. Rev. Lett. **80**, 2193 (1998); references there-in.

[10] Sung-Ho Suck Salk and S.-S. Lee, Physica B, in press(cond-mat/9907226).

[11] L. Ioffe and A. Larkin, Phys. Rev. B **39**, 8988 (1989).

[12] N. Nagaosa and P. A. Lee, Phys. Rev. B **45**, 966 (1992).

Fig. 1 The doping dependence of the superfluid weight $\frac{n_s}{m^*}$ in the U(1) slave-boson theory at $T/t = 0.001$ with $J/t = 0.2$. p is the hole doping rate.

Fig. 2 The plot of $\frac{n_s(T=0.001t)}{m^*}$ vs T_c in the U(1) slave-boson theory with $J/t = 0.2$. The thick arrows point towards increasing doping rate. $p_o = 0.07$ is the predicted optimal doping rate.

Fig. 3 The doping dependence of the superfluid weight $\frac{n_s}{m^*}$ in the SU(2) slave-boson theory at $T/t = 0.001$ with $J/t = 0.2$. p is the hole doping rate.

Fig. 4 The plot of $\frac{n_s(T=0.001t)}{m^*}$ vs T_c in the SU(2) slave-boson theory with $J/t = 0.2$. The thick arrows point towards increasing doping rate. $p_o = 0.13$ is the predicted optimal doping rate.
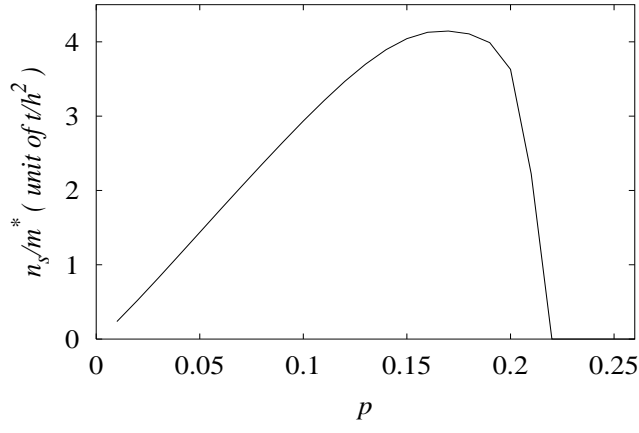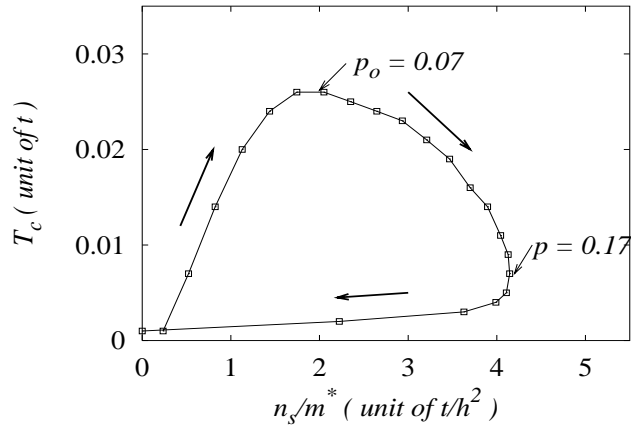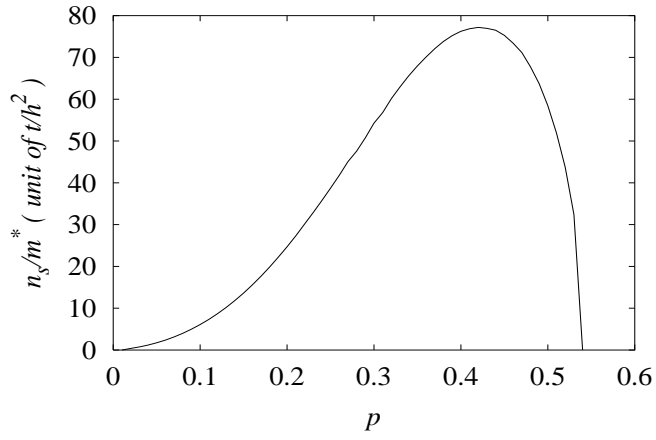
FIG. 1.

FIG. 2.

FIG. 3.

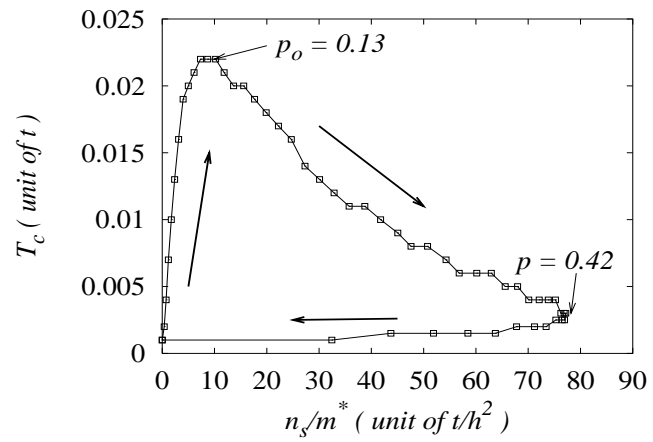FIG. 4.